BSP $\boxed{5}$ (Gr. 2)

$$y'' + (4-\alpha)y' - 4\alpha y = -3\alpha x$$

Char. Polyn.: $\lambda^2 + (4-\alpha)\lambda - 4\alpha = 0 \Rightarrow \lambda_{1,2} = \frac{-(4-\alpha) \pm \sqrt{(4+\alpha)^2}}{2} = \begin{cases} -4 \\ \alpha \end{cases}$

$$\left.\begin{aligned} y_p(x) &= Ax + B \\ y_p'(x) &= A \\ y_p''(x) &= 0 \end{aligned}\right\} \text{in Dgl} \Rightarrow 0 + (4-\alpha)A - 4\alpha(Ax+B) = -3\alpha x$$

$$-4A = -3 \Rightarrow \boxed{A = \frac{3}{4}}$$

$$3 - \frac{3\alpha}{4} - 4\alpha B - = 0 \Rightarrow \boxed{B = \frac{3}{4\alpha} - \frac{3}{16}}$$

$\alpha \neq 0$!

3 Fälle:

1.) $\alpha = -4$ (innere Resonanz)

$$\boxed{y = (C_1 + C_2 x)e^{-4x} + \frac{3}{4}x - \frac{3}{8}}$$

2.) $\alpha = 0$ („äußere Resonanz", aber hom. Glg.)

$$\boxed{y = C_1 + C_2 e^{-4x}}$$

3.) $\alpha \notin \{0,-4\} \Rightarrow \boxed{y = C_1 e^{\alpha x} + C_2 e^{-4x} + \frac{3}{4}x + \frac{3}{4\alpha} - \frac{3}{16}}$

(nur 1 Fall betrachtet ist zu wenig!!)

Bsp $\boxed{6}$ (Gr. 1) $\quad y_p = ax^b, \; y_p' = abx^{b-1}$

$\overset{\text{in Dgl.}}{\Longrightarrow} \quad abx^{b-1} + (3x - ax^b)^2 = 3 \rightsquigarrow$ brauchen x^0, also $b \in \{0, 1\}$ ↯ ✓

$\underline{b=1} \Rightarrow a + (3x - ax)^2 = 3 \Rightarrow \underline{a = 3}$

$\boxed{y_p = 3x}$ Ans: $\left.\begin{aligned} y &= u + 3x \\ y' &= u' + 3 \end{aligned}\right\} \overset{\text{in Dgl.}}{\Longrightarrow} u' + \cancel{3} + (\cancel{3x} - u - \cancel{3x})^2 = \cancel{3}$

$$u' + u^2 = 0$$
$$\Downarrow$$
$$\frac{-du}{u^2} = dx$$
$$\frac{1}{u} = x + C$$
$$u = \frac{1}{x+C}$$

$$y = \frac{1}{x+C} + 3x$$

AW: $7 = \frac{1}{C} \Rightarrow C = \frac{1}{7}$

$$\boxed{y = \frac{1}{x + 1/7} + 3x} = \frac{7}{7x+1} + 3x$$
$$= \frac{7 + 3x + 21x^2}{7x+1}$$

Bsp $\boxed{7}$ (Gr. 3)

$$\underbrace{\ddot{u}-\dot{u}}_{x^2y''} + \underbrace{2\dot{u}}_{xy'} - \underbrace{6u}_{y} = 4\underbrace{e^{t/2}}_{\sqrt{x}}$$

$u_p = Ae^{t/2}$

$\dot{u}_p = \frac{A}{2}e^{t/2}$

$\ddot{u}_p = \frac{A}{4}e^{t/2}$

$$\lambda^2 + \lambda - 6 = 0 \Rightarrow \lambda_{1,2} = \{2, -3\}$$

$$\frac{A}{4} + \frac{A}{2} - 6A = 4 \Rightarrow \boxed{A = \frac{-16}{21}}$$

$$u(t) = c_1e^{2t} + c_2e^{-3t} - \frac{16}{21}e^{t/2}$$

$$\boxed{y(x) = c_1x^2 + \frac{c_2}{x^3} - \frac{16}{21}\sqrt{x}}$$

Bsp. $\boxed{8}$ (Gr. 4)

$$\ddot{x} + \left(\frac{2}{t} - 6\right)\dot{x} + \left(9 - \frac{6}{t}\right)x = 0 \qquad x_p = e^{at}$$

$$a^2e^{at} + \left(\frac{2}{t} - 6\right)ae^{at} + \left(9 - \frac{6}{t}\right)e^{at} = 0$$

Koeff. Vgl.: konst $\cdot e^{at}$: $a^2 - 6a + 9 = 0$ ✓

$\frac{1}{t}\cdot e^{at}$: $2a - 6 = 0 \longrightarrow \underline{a = 3}$ ✓

$$\rightarrow \underline{x_1 = e^{3t}}$$

$$\underline{x_2} := e^{3t}\int \frac{e^{-\int\left(\frac{2}{t}-6\right)dt}}{e^{6t}}dt = e^{3t}\int\frac{e^{-2\ln t + \cancel{6t}}}{\cancel{e^{6t}}}dt$$

$$= e^{3t}\int\frac{dt}{t^2} = \underline{-e^{3t}\cdot\frac{1}{t}}$$

$$\Rightarrow \boxed{x(t) = c_1e^{3t} - \frac{1}{t}c_2e^{3t}} = \left(\tilde{c}_1 + \frac{1}{t}\tilde{c}_2\right)e^{3t}$$