① Gr. 2 AWP. $\boxed{(y'-6)y = 1-8x \quad , y(1) = 2}$

$$\hookrightarrow y' = \frac{1-8x+6y}{y} \qquad \boxed{\text{Sub. } \begin{array}{l} x = \bar{x} + 1/8 \\ y = \bar{y} \end{array}} \Rightarrow \bar{y}' = \frac{-8\bar{x}+6\bar{y}}{\bar{y}}$$

$$\boxed{\text{Sub. } \bar{y} = \bar{z}\cdot\bar{x}} \Rightarrow \bar{z}'\bar{x} + \bar{z} = \frac{-8+6\bar{z}}{\bar{z}} \Rightarrow \bar{z}'\bar{x} = \frac{-8+6\bar{z}-\bar{z}^2}{\bar{z}}$$

$$\xrightarrow{\text{T.d.V.}} \frac{-\bar{z}\,d\bar{z}}{\bar{z}^2-6\bar{z}+8} = \frac{d\bar{x}}{\bar{x}}$$

Partialbruch: $\frac{-\bar{z}}{\bar{z}^2-6\bar{z}+8} = \frac{A}{\bar{z}-2} + \frac{B}{\bar{z}-4}$

$-\bar{z} = A(\bar{z}-4) + B(\bar{z}-2) \Rightarrow \begin{cases} B = -2 \\ A = 1 \end{cases}$

($\bar{z}=4$ bzw $\bar{z}=2$ gesetzt)

$$\int \frac{d\bar{z}}{\bar{z}-2} - \int \frac{2\,d\bar{z}}{\bar{z}-4} = \int \frac{d\bar{x}}{\bar{x}}$$

$$\ln|\bar{z}-2| - 2\ln|\bar{z}-4| = \ln|\bar{x}| + \ln|C| \stackrel{\text{exp.}}{\Rightarrow} \frac{\bar{z}-2}{(\bar{z}-4)^2} = C\bar{x}$$

Rücksubst. $\frac{\bar{x}^2(\bar{y}-2\bar{x})}{\bar{x}(\bar{y}-4\bar{x})^2} = C\bar{x} \Rightarrow \frac{(\bar{y}-2\bar{x})}{(\bar{y}-4\bar{x})^2} = C$

Rücksubst. $\frac{y-2x+1/4}{(y-4x+1/2)^2} = C \xrightarrow[C=1/9]{\text{AWP}} \boxed{\frac{y-2x+1/4}{(y-4x+1/2)^2} = \frac{1}{9}}$

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② Gr. 1 $\boxed{(x^2+1)y' = 2xy(1-x^2y^2)}$ [Bernoulli-Dgl.]

$$y' - \frac{2x}{x^2+1}y = \frac{-2x^3}{x^2+1}y^3 \quad /\cdot(-2y^{-3}) \qquad \boxed{\text{Sub. } \begin{array}{l} z = y^{-2} \\ z' = -2y^{-3}y' \end{array}}$$

$$-2y^{-3}y' + \frac{4x}{x^2+1}y^{-2} = \frac{4x^3}{x^2+1}$$

$$z' + \frac{4x}{x^2+1}z = \frac{4x^3}{x^2+1} \quad \text{(lin. Dgl.)}$$

N.R.: $\int\frac{4x\,dx}{x^2+1} = 2\ln|x^2+1|$

$$\Rightarrow z(x) = e^{-\int\frac{4x\,dx}{x^2+1}}\left[\int\frac{4x^3}{x^2+1}e^{\int\frac{4x\,dx}{x^2+1}}dx + C\right]$$

$$= \frac{1}{(x^2+1)^2}\cdot\left[\int\frac{4x^3}{x^2+1}(x^2+1)^2\,dx + C\right]$$

$$= \frac{1}{(x^2+1)^2}\cdot\left[\int(4x^5+4x^3)dx + C\right] = \frac{1}{(x^2+1)^2}\cdot\left[\frac{2x^6}{3} + x^4 + C\right]$$

$$\Rightarrow \boxed{y(x) = \frac{1}{\sqrt{z(x)}} = \frac{x^2+1}{\sqrt{\frac{2}{3}x^6+x^4+C}}}$$

③ Gr. 3 $\quad (x^2y+2xy^2-1)\,dx+(x^3-1)\,dy=0 \qquad \mu(x+y)$!

$$\frac{N_x-M_y}{M-N}=\frac{3x^2-x^2-4xy}{x^2y+2xy^2-x^3}=\frac{2x-4y}{xy+2y^2-x^2}=\frac{2(x-2y)}{-(x+y)(x-2y)}=\boxed{\frac{-2}{x+y}}$$

N.R.: $x^2-xy-2y^2=0$

$\rightarrow x_{1,2}=\frac{y}{2}\pm\sqrt{\frac{y^2}{4}+\frac{8y^2}{4}}=\begin{cases}2y\\-y\end{cases}$

$(x+y)(x-2y)$

$$\mu(x+y)=e^{\int\frac{-2d(x+y)}{x+y}}=e^{\int\frac{-2dv}{v}}=e^{-2\ln|x+y|}=\frac{1}{(x+y)^2}$$

$$\int_0^x\frac{\tilde{x}^2y+2\tilde{x}y^2-1}{(\tilde{x}+y)^2}\,d\tilde{x}+\int\frac{-1}{y^2}\,dy=\int_0^x\left(y-\frac{1+y^3}{(\tilde{x}+y)^2}\right)dx-\int\frac{dy}{y^2}=\ldots$$

N.R.: $(x^2y+2xy^2-1):(x^2+2xy+y^2)=y-\frac{1+y^3}{(x+y)^2}$

$\underline{-(x^2y+2xy^2+y^3)}$

$\quad 0 \qquad 0 \qquad -1-y^3$

$$\Rightarrow\ \ldots=\left[\tilde{x}y+\frac{1+y^3}{\tilde{x}+y}\right]_0^x+\frac{1}{y}=\boxed{xy+\frac{1+y^3}{x+y}-\cancel{\frac{1}{y}}-y^2+\cancel{\frac{1}{y}}=C}$$

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④ Gr. 4: Orth. Trajektorien zu $\boxed{(x^2+y^2)^2+2a^2(x^2-y^2)=0}$ a … Scharparam.

$$2a^2=\frac{(x^2+y^2)^2}{y^2-x^2}$$

$\frac{\partial}{\partial x}$: $2(x^2+y^2)\cdot(2x+2yy')+2a^2(2x-2yy')=0$

einsetzen $\Longrightarrow 2\cancel{(x^2+y^2)}(2x+2yy')+\frac{(x^2+y^2)^{\cancel{2}}}{y^2-x^2}(2x-2yy')=0$ (a ist elim.)

$\Leftrightarrow (2x+2yy')(y^2-x^2)+(x^2+y^2)(x-yy')=0$

$2xy^2-2x^3+2y^3y'-2x^2yy'+x^3+xy^2-x^2yy'-y^3y'=0$

$3xy^2-x^3=(3x^2y-y^3)y' \quad\Rightarrow\quad y'=\frac{3xy^2-x^3}{3x^2y-y^3}$

ORTH. TRAJEKTORIEN, $y'\Leftrightarrow-\frac{1}{y'}$

$$y'=\frac{3x^2y-y^3}{x^3-3xy^2}\ \xRightarrow{\text{vgl. ①}}\ z'x+z=\frac{3z-z^3}{1-3z^2}\ \Rightarrow\ z'x=\frac{2z+2z^3}{1-3z^2}\quad \text{(T.d.V.)}$$

$$\int\frac{1-3z^2}{z(1+z^2)}=\int\frac{2dx}{x}\ \Rightarrow\ \ln|z|-2\ln|1+z^2|=2\ln|x|+\ln|C| \quad |\exp.$$

$$\frac{A}{z}+\frac{Bz+C}{1+z^2}\ \overset{\ldots}{\Longrightarrow}\ \frac{1}{z}-\frac{4z}{1+z^2}$$

$1-3z^2=A(1+z^2)+Bz^2+Cz$

$$\frac{z}{(1+z^2)^2}=Cx^2\ \Rightarrow\ \frac{x^4y}{x(x^2+y^2)^2}=Cx^2$$

$$\Longrightarrow \boxed{xy=C(x^2+y^2)^2}$$