Operations Research WS 2000/2001

Second exercise sheet

6. Consider the graph G in Figure 1. Assume that every edge can be traversed in both directions and that the length of the edge is the same in both directions. Use a dynamic optimization algorithm to determine the shortest paths from all nodes i, $i = 1, 2, 3, 4$, to node 5 in G.

7. The sales manager of a publisheing company for academic textbooks has hired 6 sales agents to cover the needs of three different regions of the country. The manager has decided to assign at least one sales agent to every region and that every sales agent should cover the needs of only one region. The manager wants te determine how many sales agents to assign to each of the regions such that the profit is maximized. Table 1 specifies the profit resulting by assigning 1,2,3 or 4 sales agents to any of the three regions. Solve this problem by means of dynamic optimization techniques.

number of	region		
sales agents	1	2	3
1	4	3	5
2	6	6	7
3	9	8	10
4	11	10	12

Table 1: Data for exercise nr. 7

8. A technical problem has to be solved in order to make a space project successful and to ensure a secure flight of the man to Mars. Currently there are three research teams working on that problem. In the given circumstances it has been estimated that the research teams - denoted by 1, 2 and 3 - will fail on solving the problem with a probability of 0.4, 0.6 and 0.8, respectively. As the goal is the minimization of the probability of a failure of all teams two top researchers are to be involved in the project. Table 2 specifies the probabilities that the teams will still fail in solving the problem in the case that they are extended by 0, 1 or 2 new researchers, respectively. The problem consists on determining how to assign the two top researchers to the three teams so as to minimize the probability that all teams will fail. Solve this problem by means of dynamic optimization techniques.

number	failure probability		
of new	Team		
researchers	1	2	3
0	0.4	0.6	0.8
1	0.2	0.4	0.5
2	0.15	0.2	0.3

Table 2: Data for exercise nr. 8

9. The work-load in a local company varies a lot with the seasons of the year. As it is hard to fire the workers and as training courses are very expensive, the organizative manager would not like to fire any workers during the saisons where the load is not so high. On the other hand the manager would not like to have the same high pay roll during the above mentioned “quiet” seasons as during the high load seasons, either. In principle he is also against a solution which would let the workers work more than 8 hours a day on a regular basis. Moreover, all the work is done depending on the demands of the market and hence it is impossible to produce more in “quiet seasons” and save the products for the seasons with a higher work load. Thus the manager is in a dilemma about the employment policy he should follow. The estimated number of workers needed in each season of the year is given in the following table.

Season	Spring	Summer	Automn	Winter	Spring
Nr. of workers needed	255	270	240	200	255

It is not allowed to employ less workers than the numbers specified in the above table. Hiring more workers would lead to overcosts of approximately \$ 2000/Worker/Season. It has been estimated that the overall cost of a change in the number of the employed workers from one season to the other is given as the square of the difference in the number of workers multiplied by 130. As part-time workers are allowed we might have non-integer numbers of employed workers and the costs may be fractional as well. The manager wants to determine the number of the employed workers for each season so as to minimize the overall overcosts. Solve this problem by means of dynamic optimization techniques.

10. Consider the linear program:

$$\begin{array}{ll} \max & z = 3x_1 + 2x_2 \\ \text{s.t.} & \\ & x_1 + 2x_2 \leq 6 \\ & 3x_1 + x_2 \leq 8 \\ & x_1 \geq 0,\ x_2 \geq 0 \end{array}$$

Solve this problem by means of dynamic optimization techniques.

11. Consider the following integer nonlinear program:

$$\begin{array}{ll} \max & z = x_1 x_2^2 x_3^3 \\ \text{s.t.} & \\ & x_1 + 2x_2 + 3x_3 \leq 10 \\ & x_1 \geq 1,\ x_2 \geq 1,\ x_3 \geq 1 \\ & x_1,\ x_2,\ x_3 \text{ ganzzahlig} \end{array}$$

Solve this problem by means of dynamic optimization techniques.

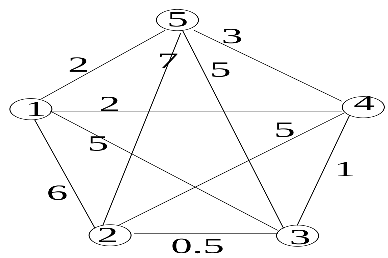

Figure 1