Game Theory WS 2013/2014

5. Exercise Sheet

- 18. Consider a mixed extension of a game in strategic form with a finite number of players $n \in \mathbb{N}$ and finite set S_i of pure strategies for every player $i \in \{1, 2, ..., n\}$. Denote by Σ_i the set of mixed strategies of player $i, i \in \{1, 2, ..., n\}$. Show that Σ_i is a convex set and its extreme points are the pure strategies $s_j^{(i)} \in S_i$, i.e. every mixed strategy $\sigma \in \Sigma_i$ can be given as a convex combination of some pure strategies from S_i and no pure strategy $s_j^{(i)} \in S_i$ can be obtained as a convex combination of strategies in Σ_i different from $s_i^{(i)}$ itself.
- 19. Write out the mixed extension for the following game, where Player I is the row player and Player II is the column player

	\mathbf{L}	Μ	R
Т	$1,\!1$	$0,\!2$	2,0
В	0,0	$1,\!0$	-1,3

Compute all equilibria in pure and mixed strategies for this game.

20. Prove that the only equilibrium in the following three player game, where Player I chooses a row, (T or B), Player II chooses a column (L or R) and Player III chooses a matrix (W or E), is (T, L, W).

W				Ε		
	\mathbf{L}	R		\mathbf{L}	R	
Т	$1,\!1,\!1$	$0,\!1,\!3$	Т	$3,\!0,\!1$	$1,\!1,\!0$	
В	$1,\!3,\!0$	$1,\!0,\!1$	В	$0,\!1,\!1$	0,0,0	

Guidance: First check whether there are equilibria in pure strategies. Then check whether there are equilibvria in which two players play pure strategies, while the third player plays a completely mixed strategy (meaning a strategy in which each one of its two pure strategies is chosen with positive probability). After that, check whether there are equilibria in which one player plays a pure strategy and the other two play completely mixed strategies. Finally check whether there are strategies in which all the players play completely mixed strategies.