Game Theory WS 2013/2014

3. Exercise Sheet

10. Consider the following two-players zero-sum game and find out if its value exists. If the value exists, then determine it, and find all the optimal strategies for each player.

	a	b	c	d
A	3	3	5	12
B	7	5	6	13
C	4	2	3	0

11. Partnership game
Lee (Player I) and Julie (Player II) are business partners. Each of the partners has to determine the amount of effort she or he will put into the business, which is denoted by e_i, $i = 1, 2$, and may be any nonnegative real number. The cost of effort e_i for Player i ist ce_i where $c > 0$ is equal for both players. The success of the business depends on the amount of effort put in by the players: the business profit is denoted by $r(e_1, e_2) = e_1^{\alpha_1} e_2^{\alpha_2}$, where $\alpha_1, \alpha_2 \in (0, 1)$ are fixed constants known by Lee and Julie, and the profit is shared equally between the two partners. Each players utility is given by the difference between the share of the profit received by that player and the cost of the effort she or he put into the business.

    (a) Describe the situation as a strategic form game. Note that the set of strategies of each player is the continuum.
    (b) Find all the Nash equilibria of the game.

12. Braess Paradox

    There are two main roads connecting San Francisco (SF) and San Jose (SJ), a norther road via Mountain View and a southern road via Cupertino. Travel time on the roads depends on the number x of cars using the road per minute, as indicated in Figure 1. For example the travel time between Mountain View and SJ is $51 + 0.1x$, where x is the number of cars pro minute using the road connecting those two cities. Each driver chooses which road to take in going from SF to SJ, with the goal of reducing to a minimum the amount of travel time. Early in the morning, 60 cars per minute get on the road from SF to SJ (where we assume the travellers leave early enough in the morning so that they are the only ones in the road at that hour).

    (a) Describe this situation as a strategic form game, in which each driver chooses the route he will take.
    (b) Determine all the Nash equilibria of this game? How much time does the trip take at an early morning hour under these equilibria?
    (c) The California Department of Transportation constructs a new road between Mountain View and Cupertino, with travel time between these cities $10 + 0.1x$. This road is one way, enabling travel solely from Mountain View to Cupertino.
    Find a Nash equilibrium in the new game. Under this equilibrium how much time does it take to get to SJ from SF at an early morning hour?
    (d) Does the construction of the additional road improve travel time?

13. The classical Bertrand game is the following. Assume that n companies which produce the same product are competing for customers. If each company i has a production level of q_i, there will be $q = \sum_{i=1}^{n} q_i$ unit of products on the market. Assume that the demand for this product depends on the price and if q units are on the market, then price will settle so that all q units are sold. Assume that we are given a “demand-price-curve” $p(d)$, which gives the price at which all $p(d)$ units can be sold. Assume that $p(d)$ is a monotone decreasing, differentiable function of d. With this definition the income of the firm i will be $q_i p(q)$. Assume that production is very cheap and each firm will produce to maximize its income.

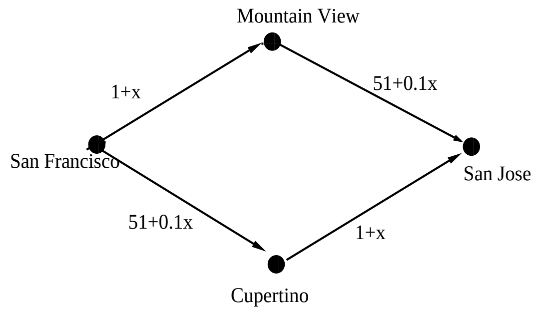

Figure 1: Scheme for Exercise 12

(a) Show that the total income for a monopolistic firm, can be arbitrarily higher than the total income of many different firms sharing the same market. Hint: this is true for almost all price curves; you may want to use, e.g., $p(d) = 1 - d$.

(b) Assume that $p(d)$ is twice differentiable, monotone decreasing, and $p''(d) \leq 0$. Show that the monopolistic income is at most n times the total income of the n competing companies.

14. (For ambitious students interested in algorithmic topics; can affect the total evaluation score only as a bonus.)

Consider an n person game in which each player has only 2 strategies. There are 2^n possible strategy vectors in this game, therefore the game in matrix form is exponentially large. We consider a special class of this kind of games, so called *graphical games*, where the payoff of a player can depend only on the strategies of a subset of players. We define a dependence graph G, whose nodes are the players and an edge between two players i and j represents the fact that the payoff of player i depends on the strategy of the player j and vice versa. Thus, if node i has k neighbors in G, then its payoff depends only on its own strategy and the strategies of its k neighbors.

Consider a game where the players have two strategies each and the dependence graph G is a tree with maximum degree 3. Give a polynomial time algorithm to decide if such a game has a Nash equilibrium. (Recall that there are 2^n possible strategy vectors, yet your algorithm must run in time polynomial in n.)