Game Theory WS 2013/2014

13. Exercise Sheet

48. A player i in a coalitional game is a null player if for every coalition S,

$$v(S \cup \{i\}) = v(S)$$
 holds.

Show that, if the core is nonempty, then $x_i = 0$ for every imputation x in the core and every null player i.

- 49. A player *i* in a simple game is a veto player if v(S) = 0 for every coalition S that does not contain *i*.
 - (a) Show that the core of a simple game (N; v) satisfying v(N) = 1 contains every imputation x satisfying $x_i = 0$ for every player i which is not a veto player, and does not contain any other imputations. In other words, the only imputations in the core are those in which the set of veto players divide the worth of the grand coalition V(N) between them.
 - (b) Consider a simple majority game (N; v) with n := |N| in which a coalition wins if and only if it has at leas $\frac{n+1}{2}$ votes; that is, for every coalition $S \subseteq N$, the following equality holds:

$$v(S) = \begin{cases} 1 & \text{if } |S| \ge \frac{n+1}{2} \\ 0 & \text{if } |S| < \frac{n+1}{2} \end{cases}$$

What is the core of this game?

- (c) What is the core of a simple coalitional game without veto players?
- 50. Define the dual game of a coalitional game (N; v) to be the coalitional game $(N; v^*)$, where

$$v^*(S) = v(N) - v(N \setminus S) , \forall S \subseteq N$$

Is the core of (N; v) nonempty if and only if the core of $(N; v^*)$ is nonempty? Either prove this claim or provide a counterexample.

51. Prove that the coalitional game $(\{1, 2, 3, 4\}; v)$ in which v is defined by

$$v(S) = \begin{cases} 0 & \text{if } |S| = 1\\ 30 & \text{if } |S| = 2\\ 0 & \text{if } |S| = 3\\ 50 & \text{if } |S| = 4 \end{cases}$$

has an empty core.

Guidance: Find a balanced collection of coalitions which does not fulfil the Bondareva-Shapley condition.

- 52. (a) Prove that the coalitional game (N; v), where $N = \{1, 2\}$ and the coalitional function is defined by $v(\{1\}) = v\{2\}) = 1$, $V(\{1, 2\}) = 3$, is a market game.
 - (b) Find a market such that (N; v) is the game derived from it.