Operations Research
Winter term 2015/2016

5th work sheet (inventory models)

24. Consider an inventory system with three products in inventory. The fixed ordering costs K_i, the variable ordering costs c_i per piece, the inventory costs h_i per piece and month, and the demands rates μ_i for product i, $i \in \{1, 2, 3\}$, are given in the following table:

	Product 1	Product 2	Product 3
K (in Euro)	40	10	45
c (in Euro per piece)	0.5	1	0.25
h (in Euro per piece and month)	0.1	0.2	0.05
μ (in pieces per month)	200	100	50

   (a) Assume that the cycle length should be the same for all products, such that all products can be delivered at the same time. Determine the optimal lot sizes, the optimal cycle length and the optimal (i.e. minimal) overall ordering and inventory costs per product.

   (b) Assume that the value of the products in inventory should not exceed a certain amount γ. Determine the optimal lot sizes, the optimal cycle length and the optimal (i.e. minimal) overall ordering and inventory costs per product for $\gamma = 200$ Euro, $\gamma = 150$ Euro and $\gamma = 30$ Euro, respectively.

25. Assume that the demand on a particular spare part of an old aeroplane type is exponentially distributed eith expected value equal to 50. The production of the old aeroplane type will be shut down in one year of time and thus the whole production of spare parts which will be eventually needed should ideally happen during the current year. The variable production costs amout to 1000 Euro per piece if the production happens in the curent year. In the case that the production happens at a later point in time, the variable production costs amout to 10000 Euro per piece. There are no fixed production costs. The inventory costs for the spare sparts remaining in inventory at the end of the year amount to 300 per piece.

   (a) Determine the optimal number of the spare parts to be produced such that the expected overall costs are minimized (with the settings of a standard one period model inclding a starting invetory level of 0 pieces).

   (b) Determine the optimal number of the spare parts to be produced such that the expected overall costs are minimized if the initial inventory level is 23 pieces.

   (c) Assume that the shortage costs cannot be estimated yet at the moment. How many spare parts should be produced such that a shortage will be realized with probability of at most 0.1 at the end of the one year period.

   (d) Consider the original standard model including shortage costs. Assume that the lot size computed in point (c) is optimal for this model and compute the shortage costs per piece and year such that this assumption holds.

26. The stochastic one-period inventory model is extended as follows. There are two products in inventory. They can serve as a substitute for each-other, i.e. Product 1 (Product 2) can be used to fulfill the demand on Product 2 (Product 1), in the case that the demand cannot be fulfilled by the inventory of the specific product. Assume for simplicity that the inventory level of each product equals 0 at the beginning of the planning horizon. Moreover assume that both the fixed ordering costs and the costs of missing demand equal 0. Let c_i be the variable ordering cost per unit of product i and let s_i be the profit realized by fulfilling the demand of one unit of product i, $i = 1, 2$. Hence the fulfillment of a demand for z units of product i results in a net profit of $z(s_i - c_i)$, $i = 1, 2$, for the inventory holder.

Let X_i be a random variable representing the demand on product i and let f_i be the probability density function of X_i, $i = 1, 2$. Further let $a_i \in (0, 100)$ be the percentage of customers which would eventually accept a substitute for their demand. Denote by $G(u_1, u_2)$ the (random) profit of the inventory holder if he orders u_i units of product i, $i = 1, 2$. The goal is to determine the amounts u_i of products to be ordered, $i = 1, 2$, such that the expected profit $E(G(u_1, u_2))$ is maximized. Derive a formula for $E(G(u_1, u_2))$.

27. Consider a three-period stochastic stationary inventory model as the one dealt with in the lecture (version A and B). Let the random demand R be uniformly distributed over the interval $[0, 10]$. The other input parameters of the problem are as follows: $c = 1$ Euro per product unit, $K = 5$ Euro, $h = 3$ Euro per product unit and period, $p = 4$ Euro per product unit and period, and $\alpha = 0.9$. Determine an optimal ordering policy $z_j^*(x)$, $x \in \mathbb{R}$, $j = 1, 2, 3$, and the corresponding costs, for each of the models A and B (cf. lecture). Determine also the lower and upper bounds for the ordering point and the ordering level in every period.

28. Consider a stochastic stationary inventory model with an infinite number of periods as described in the lecture. Let the random demand R be uniformly distributed over the interval $[0, 10]$. The other input parameters of the problem are as follows: $c = 1$ Euro per product unit, $K = 0$ Euro, $h = 3$ Euro per product unit and period, $p = 4$ Euro per product unit and period, and $\alpha = 0.9$. Determine an optimal ordering policy $z^*(x)$, $x \in \mathbb{R}$ and the corresponding costs.

29. Consider the following multicriteria optimization problem (MCOP) $(X, f, \mathbb{R}^2)/\text{id}/(\mathbb{R}^2, <)$, where $X = [-1, 1] \subseteq \mathbb{R}$, $f = (f_1, f_2)$ with $f_1(x) = \sqrt{5 - x^2}$, $f_2(x) = x/2$, and $<$ is the *component-wise order* in $\mathbb{R}^2$. Give a graphical representation of the sets X and $Y = f(X)$ of feasible solutions in the decision space and in the objective function space, respectively. Determine the Pareto-set $X_{Par} \subseteq X$ and the efficient set $Y_{eff} \subseteq Y$.

30. Solve the problem from Exercise 29 if the component-wise order is substituted by the *max order* or the *lexicographic order*, respectively:

$$\min_{x \in [-1,1]} \max_{i=1,2} f_i(x)$$

$$lex \min_{x \in [-1,1]} (f_1(x), f_2(x))$$

$$lex \min_{x \in [-1,1]} (f_2(x), f_1(x))$$

Compare the set of the optimal solutions in any of three cases above to the set of the Pareto-optimal solutions to the original problem.