Operations Research
Winter term 2015/2016

4th work sheet (inventory models)

18. MBI is a manufacturer of personal computers. All of its personal computers use a certain device driver which it purchases from YNOS. MBI operates its factory 52 weeks per year and assembles 100 pieces of the device drivers mentioned above into the computers per week. MBIs inventory cost rate is 20% of the value of the inventory (based on purchase cost). Regardless of the order size, the administrative cost of placing an order with YNOS has been estimated to be 1440 Euro. A quantity discount is offered by YNOS for large orders as shown below, where the price $c(Q)$ of one device driver is specified depending on the order quantity Q.

$$c(Q) = \begin{cases} 100 & \text{Euro per unit if } 1 \leq Q \leq 99 \\ 95 & \text{Euro per unit if } 100 \leq Q \leq 499 \\ 90 & \text{Euro per unit if } Q \geq 500 \end{cases}$$

Determine the optimal order quantity according to the EOQ model with quantity discounts. What is the resulting total cost per year? How many orders have to be placed per year and what is the time interval between two consecutive orders?

19. Consider the deterministic dynamic inventory model discussed in the lecture (in Section 2.2.) and the statement of the lemma “In each period i, either the inventory level at the beginning of the period is zero and the order size is positive, i.e. $x_i = 0$ and $u_i > 0$, or the inventory level at the beginning of the period is positive and the order size is zero, i.e. $x_i > 0$ and $u_i = 0$.”. Is this statement true if the assumptions of the problem are modified as follows:

    (a) The inventory cost coefficient depends on the period, i.e. the inventory of one unit of product for one unit of time costs h_i in period i, $i = 1, 2, \ldots, n$.

    (b) The fixed ordering cost depends on the period., i.e. placing a (positive) order in period i implies a fixed ordering cost K_i, $1 = 1, 2, \ldots, n$.

    (c) The variable ordering cost depends on the period., i.e. one unit of product ordered in period i costs c_i, $1 = 1, 2, \ldots, n$.

    (d) All the assumptions of the problem as described in the lecture hold, except for the variable ordering cost coefficient: it does not depend on the period, but it does depend on the order quantity, i.e. quantity discounts should be considered.

    In any case either prove that the statement of the lema holds or provide a counter-example.

20. Suppose that production planning is to be done for the next 5 months, where the respective demands are $r_1 = r_3 = r_4 = 2$, $r_2 = 4$, and $r_5 = 3$. The fixed ordering cost ist 4000 Euro, the variable production cost is 1000 Euro per unit, and the inventory cost is 300 Euro per unit and month. Use the algorithm of Wagner and Whittin to determine an optimal production schedule which fulfills the monthly requirements and yealds the minimum total cost.

21. Consider a situation where a particular product is produced and placed in in-process inventory until it is needed in a subsequent production process. The number of units required in each of the next three months, the setup cost and the regular-time production cost per unit (in unit of thousands of Euros) that would be incurred in each month are as follows:

Month	Requirement	Setup cost	Regular-time unit cost
1	1	5	8
2	3	10	10
3	2	15	9

There currently is 1 unit in inventory, and we want to have 2 units in inventory at the end of the 3 months. A maximum of 3 units can be produced on regular-time production in each month, although 1 additional unit can be produced on overtime at a cost that is 2 units larger than the regular-time unit production cost. The inventory cost is 1 per unit of product for each extra month that it is stored. Shortage is not allowed. Use dynamic programming to determine how many units should be produced in each month in order to minimize the total cost. Is it possible to use a similar approach to that of Wagner-and-Whittin (cf. lecture)?

22. Consider an inventory system which fits the model of a serial two-stage inventory model (cf. lecture, Section 2.3), where $K_1 = 15000$, $K_2 = 500$, $h_1 = 20$, $h_2 = 22$ and $r = 5000$. Solve this problem by optimizing both stages simultaneoulsy as discussed in the lecture. Then find a solution of this problem by optimizing the two stages separately. Compare the solutions obtained by these two different approaches and their costs, respectively.

23. Consider a five-stage inventory system which fits the serial multi-stage inventory model discussed in the lecture (cf. Section 2.4). Let the demand rate be $r = 1000$ and the other parameters of the model be given in the table below.

Stage i	K_i (in Euro)	h_i (in Euro)
1	125000	2
2	20000	10
3	6000	15
4	10000	20
5	250	30

Solve this problem by applying the approximation algorithm discussed in the lecture. Save all intermediate solutions and their corresponding overall costs in a table and compare the latter. Give a (non-trivial) lower bound for the optimal cost; this bound can be obtained by using the cost corresponding to the approximate solution.