

**Operations Research**  
**Winter term 2015/2016**

**3d work sheet (dynamic optimization and inventory models )**

15. Consider a deterministic dynamic minimization problem with an infinite planning horizon and discount factor  $\alpha \in (0, 1)$ . Let the set  $\mathcal{S}$  of the feasible states and the set  $\mathcal{A}$  of the feasible decision be given as  $\mathcal{S} = \{0, 1, 2\}$  and  $\mathcal{A} = \{0, 1, 2\}$ , respectively. Moreover let  $S_n = \mathcal{S}$ , and  $A_n(s) = \mathcal{A}$  hold for all  $n \in \mathbb{N}$  and for all  $s \in \mathcal{S}$ . Further let the state transition function and the one state cost function be given as follows:

$$z(s, a) = (s + a) \bmod 3, \quad r(s, a) = \gamma a^2 - \beta(s + 1), \text{ for all } s \in \mathcal{S}, \text{ and for all } a \in \mathcal{A},$$

where  $\gamma > 0$ ,  $\beta > 0$ , are two given parameters. Formulate some conditions to be fulfilled by the parameters  $\gamma$  and  $\beta$  such that the policy iteration procedure terminates with an optimal solution right after the second iteration. How would you comment/interpret this result?

(Consider that  $\alpha$  is a given constant in  $(0, 1)$ . The conditions on  $\beta$  and  $\gamma$  have to be specified depending on  $\alpha$ .)

16. Consider an inventory system which fulfills the general assumption discussed in the lecture if no other specification are given. The demand amounts to 650 units per week and it is constant over time. The fixed ordering cost amounts to 25 Euro per order, the variable ordering cost is 3 Euro per product unit, and the inventory cost is 0.005 Euro per product unit and week.
- (a) Consider the EOQ model where no shortage is allowed. Determine the optimal lot-size and the optimal ordering frequency so as to minimize the overall cost per time unit.
  - (b) Assume that shortage is allowed and that the shortage cost amounts to 3 Euro per product unit and week. Determine the optimal lot-size and the optimal ordering frequency so as to minimize the overall cost per time unit.
17. In the basic EOQ model, suppose that the stock is replenished uniformly (rather than instantaneously) at a rate of  $b$  product units per time unit until the order quantity  $Q$  is fulfilled. The demand is served uniformly at a rate of  $a$  product units per time unit, where  $a < b$ . Replenishment and delivery of products are made simultaneously. For example, if  $Q = 60$ ,  $b = 3$  and  $a = 2$ , then three units of product arrive each day for the days 1 to 20, 31 to 50, and so on, whereas products are delivered at a rate of 2 units per day every day.
- (a) Visualize by means of a diagram the inventory level versus time for the above mentioned example.
  - (b) Give the total cost per time unit in terms of the fixed ordering cost  $K$ , the order size  $Q$ , the variable ordering cost  $c$ , the inventory cost  $h$  and the parameters  $a$  and  $b$ .
  - (c) Determine the optimal lot-size  $Q^*$  and the optimal ordering frequency so as to minimize the overall cost per time unit.