Workshop on

Tractable special cases of hard combinatorial optimization problems

December 15-16, 2014

Department of Optimization and Discrete Mathematics
Graz University of Technology

Steyrergasse 30
8010 Graz, Austria

Program

Monday, December 15 SR C208, Steyrergasse 30, 2nd floor

8:30	Registration
8:50-9:00	Welcome
9:00-10:00	Gerhard J. Woeginger, Eindhoven University of Technology, The Netherlands *Tractable special cases through linear algebra*
10:00-10:30	Coffee Break
10:30-11:30	Frits Spieksma, KU Leuven, Belgium *Multi-index assignment problems: an overview*
11:35-12:05	Matteo Seminaroti, Centrum Wiskunde & Informatica (CWI), The Netherlands *The quadratic assignment problem is easy for Robinsonian matrices*
12:05-14:15	Lunch
14:15-14:45	Ante Ćustić, University of Zagreb, Croatia *The planar 3-dimensional assignment problem with Monge-like cost arrays*
14:50-15:20	Marie MacCaig, University of Birmingham, United Kingdom *The integer image problem in the max algebra*
15:25-15:55	Rostislav Staněk, University of Graz, Austria *A polynomial solvable case of the data arrangement problem on binary trees*
15:55-16:25	Coffee Break
16:25-17:15	Open problem session

Tuesday, December 16 SR C208, Steyrergasse 30, 2nd floor

9:00-10:00	Vladimir Deĭneko, Warwick Business School, United Kingdom *Special structures in polynomially solvable cases: Is there much in common?*
10:00-10:30	Coffee Break
10:30-11:30	Martin Milanič, University of Primorska, Slovenia *Vector connectivity in graphs*
11:35-12:05	Nina Chiarelli, University of Primorska, Slovenia *Linear separation of variants of dominating sets in graphs*
12:05-14:15	Lunch
14:15-14:45	Joachim Schauer, University of Graz, Austria *The knapsack problem on weakly chordal conflict graphs*
14:50-15:45	Open problem session

Directions to the institute

From the Graz International Airport (GRZ) to the Institute

A taxi from the Graz international airport to the institute costs approx. 25 EUR.

There are trains (S-Bahn no. 5) from the Graz airport (Train stop: Flughafen Graz-Feldkirchen) to the Graz main railway station (Train stop: Graz Hauptbahnhof), which run every half an hour. From the Graz main railway station to the institute see below.

There are less frequent buses (Bus 630) from the airport to the city centre (Bus stop: Graz Jakominiplatz). From Jakominiplatz to the institute you may walk about 10 minutes to the institute or take a tram to the stop Neue Technik (see map below). On Sunday or Monday through Saturday after 8 pm you should take tram no. 26 (shortened version of tram no. 6 which does not operate on Sundays and after 8 pm). Otherwise, Monday through Saturday until 8 pm take tram no. 6.

From the Graz Main Railway Station (Graz Hauptbahnhof) to the Institute

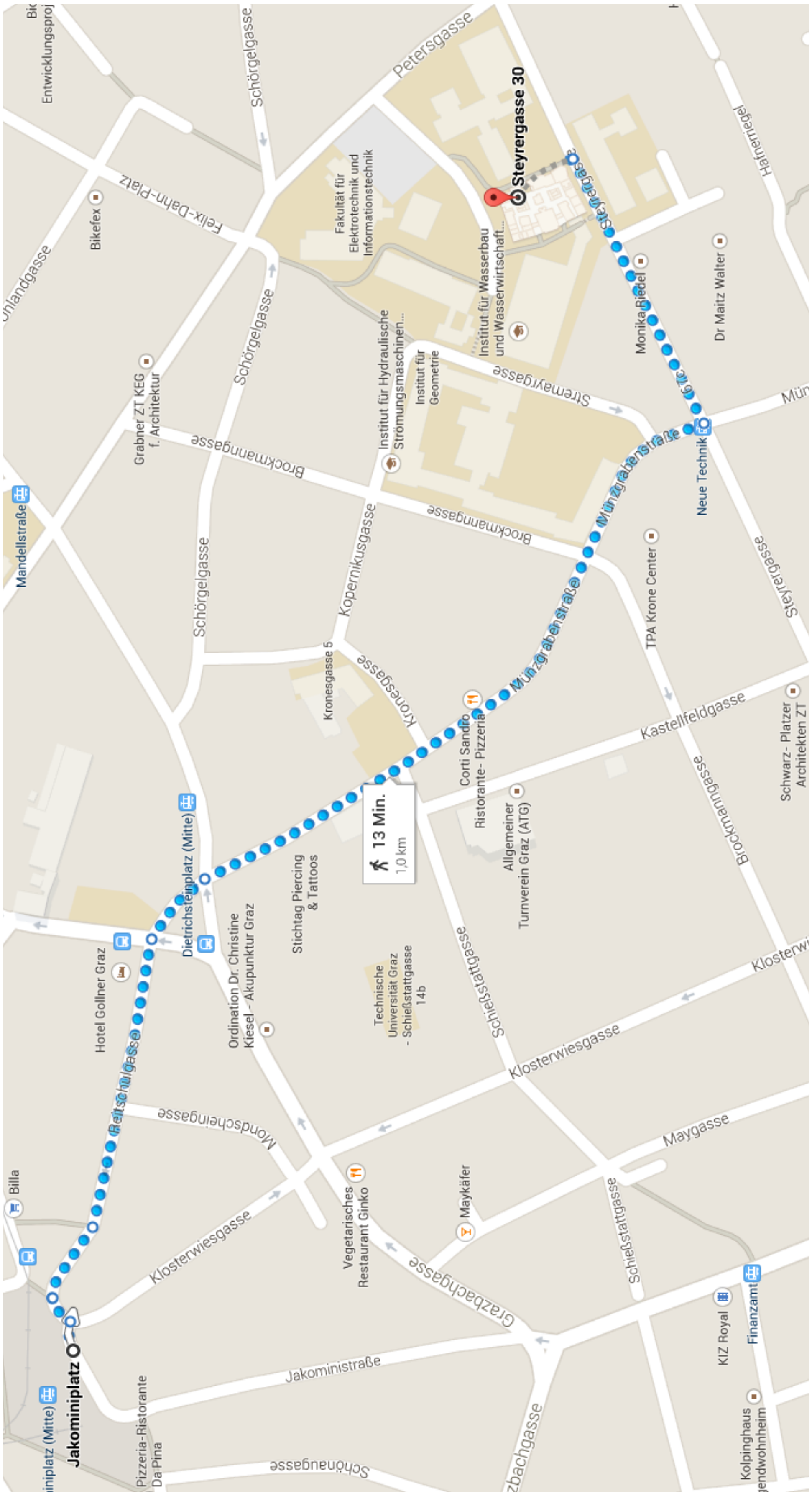
From the Graz main railway station (Tram stop: Graz Hauptbahnhof) you may take tram no. 6 to the Neue Technik stop on Monday through Saturday until 8 pm. Alternatively, you may take tram no. 1, 3 or 7 to Jakominiplatz and than walk to the institute for about 10 minutes. On Sunday or Monday through Saturday after 8 pm you make tram no. 1 or 7 at Graz Hauptbahnhof to Jakominiplatz. From Jakominiplatz to the institute you may walk for about 10 minutes or take tram no. 26 (shortened version of tram no. 6 which does not operate on Sundays and after 8 pm) to the stop Neue Technik (see map below).

A single ticket which costs 2.10 EUR is valid for all public transport services within the Graz central tariff zone 101 (including the Graz airport) for one hour. For time tables and ticket prices, please visit the Styrian Public Transport Association website (http://www.verbundlinie.at/lang/en/).

From the Vienna International Airport (VIE) to Graz

There are direct flight connections from the Vienna international airport to the Graz international airport. For further information, please check e.g. the Austrian Airlines website (http://www.austrian.com/).

If you prefer public ground transportation options for travel from the Vienna airport to Graz you may take a bus (Bus 1187 of Vienna Airport Lines/Postbus) at the Vienna airport in the direction of Wien Westbahnhof, which runs every half an hour, and get out at the first stop Wien Dörfelstraße/Meidling Bahnhof, which takes about half an hour. From the Wien Meidling railway station (Wien Meidling Bahnhof) to the Graz main railway station (Graz Hauptbahnhof) you may take a "RailJet" train bound for Graz, which takes about two and a half hours. There are other options which require more changes of buses and trains en route, but may save 15 minutes travel time. For time tables and ticket prices, please visit the Austrian Federal Railways (ÖBB) website (http://www.oebb.at/en/).



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13 Min.
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Reitschulgasse
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Steyrergasse
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Institut für Geometrie
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Brockmanngasse
Kopernikusgasse
Kronesgasse
Kronesgasse 5
Schörgelgasse
Petersgasse
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Vegetarisches Restaurant Ginko
Maykäfer
Maygasse
Grazbachgasse
Jakoministraße
Schönaugasse
Pizzeria-Ristorante Da Pina
Billa
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Abstracts

Tractable special cases through linear algebra

Gerhard Woeginger, Eindhoven University of Technology, The Netherlands

The talk surveys and discusses a number of tractable special cases of the travelling salesman problem and the quadratic assignment problem. As a common theme, the underlying cost matrices can be defined and easily understood through basic linear algebra: they form a cone, or a low-dimensional subspace, or they have bounded max-plus rank. Of course, also the algorithmic analysis uses tools from linear algebra.

Multi-index assignment problems: an overview

Frits Spieksma, KU Leuven, Belgium

In this presentation we give an overview of applications of, and algorithms for, special cases of multi-index assignment problems (MIAPs). MIAPs, and relatives of it, have a long history in combinatorial optimization, both in applications as well as in theoretical results, starting at least in the 1950's. A prominent example of a MIAP is the so-called axial three index assignment problem (3AP) which has many applications in a variety of domains including clustering and production. A description of 3AP is as follows. Given are three n-sets R, G, and B. For each triple in $R \times G \times B$ a cost-coefficient $c(i, j, k)$ is given. The problem is to find n triples such that each element is in exactly one triple, while minimizing total cost. We show positive and negative results for finding an optimal solution to this problem that depend upon different ways of how the costs $c(i, j, k)$ are specified.

The quadratic assignment problem is easy for Robinsonian matrices

Monique Laurent, Centrum Wiskunde & Informatica (CWI) and Tilburg University, The Netherlands
Matteo Seminaroti, Centrum Wiskunde & Informatica (CWI), The Netherlands

We present a new polynomially solvable case of the Quadratic Assignment Problem in Koopmans-Beckman form QAP(A, B), by showing that the identity permutation is optimal when A and B are respectively a Robinson similarity and dissimilarity matrix and one of A or B is a Toeplitz matrix. A Robinson (dis)similarity matrix is a symmetric matrix whose entries (increase) decrease monotonically along rows and columns when moving away from the diagonal, and such matrices arise in the classical seriation problem. The presentation is based on the work in [1].

References

[1] M. Laurent and M. Seminaroti. The quadratic assignment problem is easy for Robinsonian matrices. arXiv:1407.2801, 2014.

The planar 3-dimensional assignment problem with Monge-like cost arrays

Ante Ćustić, University of Zagreb, Croatia
Bettina Klinz, Graz University of Technology, Austria
Gerhard Woeginger, Eindhoven University of Technology, The Netherlands

Given an $n \times n \times p$ cost array C we consider the problem p-P3AP which consists in finding p pairwise disjoint permutations $\varphi_1, \varphi_2, \ldots, \varphi_p$ of $\{1, \ldots, n\}$ such that $\sum_{k=1}^{p} \sum_{i=1}^{n} c_{i\varphi_k(i)k}$ is minimized. For the case $p = n$ the planar 3-dimensional assignment problem (P3AP) results. Our main result concerns the p-P3AP on cost arrays C that are so called layered Monge arrays. In a layered Monge array all $n \times n$ matrices that result from fixing the third index k are Monge matrices. We prove that the p-P3AP and the P3AP remain NP-hard for layered Monge arrays. Furthermore, we show that in the layered Monge case there always exists an optimal solution of the p-P3AP which can be represented as a band matrix with bandwidth $\leq 4p - 3$. This structural result allows us to provide a dynamic programming algorithm that solves the p-P3AP on layered Monge arrays in polynomial time when p is fixed.

The integer image problem in the max algebra

Marie MacCaig, University of Birmingham, United Kingdom

The max-algebraic semiring is $(\bar{\mathbb{R}}, \oplus, \otimes)$ where $\bar{\mathbb{R}} = \mathbb{R} \cup \{-\infty\}$, $a \oplus b = \max(a, b)$ and $a \otimes b = a + b$. We discuss the problem of finding an integer vector in the image space of a matrix in the max-algebra, which we refer to as the integer image problem. Whilst the complexity of this problem is unresolved, we show that if we introduce some additional assumptions on the input matrices, we can describe a number of cases where we can determine in polynomial time whether such an integer vector exists, and find one if it does exist. On the other hand we describe related problems which are NP-hard. Our main result is that the problem of finding an integer image is equivalent to finding a special type of integer image of a matrix satisfying a property we call *column typical*. For a subclass of matrices this problem is polynomially solvable but if we remove the column typical assumption then it becomes NP-hard.

A polynomial solvable case of the data arrangement problem on binary trees

Eranda Dragoti-Çela, Graz University of Technology, Austria
Rostislav Staněk, University of Graz, Austria
Joachim Schauer, University of Graz, Austria

The data arrangement problem on regular trees (DAPT) consists in assigning the vertices of a given graph G to the leaves of a d-regular tree T such that the sum of the pairwise distances of all pairs of leaves in T which correspond to edges of G is minimised. Luczak and Noble [1] have shown that this problem is NP-hard for every fixed $d \geq 2$. The question about the computational complexity of the DAPT in the case where the guest graph is a tree is still open.

We deal with one special case of this problem where both the guest and the host graph are binary regular trees.

References

[1] M.J. Luczak and S.D. Noble, Optimal arrangement of data in a tree directory, Discrete Applied Mathematics 121 (1-3), 307-315, 2002.

Special structures in polynomially solvable cases: Is there much in common?

Vladimir Deĭneko, Warwick Business School, United Kingdom

In our talk we present a survey of polynomially solvable cases of NP-hard problems with an emphasis on common structures in these cases. We concentrate on the cases where specially structured matrices are involved. In most considered cases permuting rows and columns of specially structured matrices destroy the properties needed. We discuss the arising recognition problems and pose quite a few open questions from this exciting area of research.

Vector connectivity in graphs

Martin Milanič, University of Primorska, Slovenia

Motivated by challenges related to domination, connectivity, and information propagation in social and other networks, we study the Vector Connectivity problem. This problem, introduced by Boros et al. in [1], takes as input a graph G and an integer $r(v)$ for every vertex v of G, and the objective is to find a vertex subset S of minimum cardinality such that every vertex v either belongs to S, or is connected to at least $r(v)$ vertices of S by disjoint paths. If we require each path to be of length exactly 1, we get the well-known vector domination problem, which is a generalization of the dominating set and vertex cover problems. Consequently, the vector connectivity problem becomes NP-hard if an upper bound on the length of the disjoint paths is also supplied as input. Due to the hardness of these domination variants even on restricted graph classes, like split graphs, Vector Connectivity seems to be a natural problem to study for drawing the boundaries of tractability for this type of problems.

In the talk, I will give an overview of known complexity results for the Vector Connectivity problem. In particular, the problem can be solved in polynomial tim e on split graphs, in addition to cographs and trees. On the other hand, the problem is NP-hard for planar line graphs and for planar bipartite graphs, APX-hard on general graphs, and can be approximated in polynomial time within a factor of$\log n + 2$ on all n-vertex graphs. Vertex covers and dominating sets in a graph G can be easily characterized as hitting sets of derived hypergraphs (of G itself, and of the closed neighborhood hyper graph of G, respectively). Using Menger's Theorem, a similar characterization of vector connectivity sets can be derived.

Based on joint works with Endre Boros, Ferdinando Cicalese, Pinar Heggernes, Pim van 't Hof, and Romeo Rizzi (see [1]-[3]).

References

[1] E. Boros, P. Heggernes, P. van 't Hof, and M. Milanič. Vector connectivity in graphs. In *Theory and applications of models of computation*, volume **7876** of *Lecture Notes in Comput. Sci.*, pages 331–342. Springer, Heidelberg, 2013.

[2] E. Boros, P. Heggernes, P. van 't Hof, and M. Milanič. Vector connectivity in graphs. *Networks*, **63(4)**, 277–285, 2014.

[3] F. Cicalese, M. Milanič, and R. Rizzi. On the complexity of the vector connectivity problem. Submitted, 2014.

Linear separation of variants of dominating sets in graphs

Nina Chiarelli, University of Primorska, Slovenia
Martin Milanič, University of Primorska, Slovenia

A possible approach for dealing with the intractability of a given decision or optimiza- tion problem is to identify restrictions on input instances under which the problem can still be solved efficiently. One generic framework for describing a kind of such restrictions in case of graph problems is the following: Given a graph G, does G admit non-negative integer weights on its vertices (or edges, depending on the problem) and a set T of integers such that a subset X of its vertices (or edges) has property P if and only if the sum of the weights of elements of X belongs to T? Property P can denote any of the desired substructures we are looking for, such as matchings, cliques, stable sets, dominating sets, etc.

In general, the advantages of the above framework depend both on the choice of property P and on the constraints (if any) imposed on the structure of the set T. For example, if P denotes the property of being a stable (independent) set, and set T is restricted to be an interval unbounded from below, we obtain the class of threshold graphs [2], which is very well understood and admits several characterizations and linear time algorithms for recognition and for several optimization problems [4]. If P denotes the property of being a dominating set and T is an interval unbounded from above, we obtain the class of domishold graphs [1], which enjoys similar properties as the class of threshold graphs. On the other hand, if P is the property of being a maximal stable set and T is restricted to consist of a single number, we obtain the class of equistable graphs [6], for which the recognition complexity is open (see, e.g., [3]), no structural characterization is known, and several NP-hard optimization problems remain intractable on this class [5].

In this talk we will discuss two more graph classes that can be described within the above framework, namely the classes of total domishold and of connected domishold graphs, obtained by taking P to be the the property of being a total, resp. a connected dominating set (and letting T be an interval unbounded from above).

References

[1] Benzaken, C. and Hammer, P. L., Linear separation of dominating sets in graphs. Volume 3 of Ann. of Discrete Math., pages 1-10, North-Holland, Amsterdam, 1978.

[2] Chvátal, V. and Hammer, P. L., Aggregation of inequalities in integer programming. In Studies in integer programming (Proc. Workshop, Bonn,1975), volume 1 of Ann. of Discrete Math., pages 145-162, North-Holland, Amsterdam, 1977.

[3] Levit, V. E., Milanič, M. and Tankus, D., On the Recognition of k-Equistable Graphs. In Proc. 38th International Workshop on Graph-Theoretic Concepts in Computer Science (WG 2012), pages 286-296, 2012.

[4] Mahadev, N. V. R. and Peled, U. N., Threshold graphs and related topics, volume 56 of Ann. of Discrete Math., pages xiv+543, North-Holland, Amsterdam, 1995.

[5] Milanič, M. and Orlin, J. and Rudolf, G., Complexity results for equistable graphs and related classes. Volume 188 of Ann. Oper. Res., pages 359-370, 1978.

[6] Payan, C., A class of threshold and domishold graphs: equistable and equidominating graphs. Volume 25 of Discrete Mathematics, pages 47-52, 1980.

The knapsack problem on weakly chordal conflict graphs

Ulrich Pferschy, University of Graz, Austria
Joachim Schauer, University of Graz, Austria

We consider the classical 0-1 knapsack problem with an additional disjunctive relation on pairs of items represented by a conflict graph. This means that adjacent vertices in the graph correspond to items that may not be packed together in the knapsack. We present pseudopolynomial time algorithms for weakly chordal graphs and more generally for graph classes do do contain only polynomially many maximal cliques. It is based on results by Fomin and Villanger (2010) and Lokshtanov et al. (2014) and can be extended to obtain FPTASs.

References

F.V. Fomin and Y. Villanger. Finding induced subgraphs via minimal triangulations. In STACS, pages 383-394, 2010.

D. Lokshtanov, M. Vatshelle, and Y. Villanger. Independent set in p_5-free graphs in polynomial time. In SODA, pages 570-581, 2014.

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